

MATH 162A Review: Vector Calculus

Facts to Know:

The chain rule: Let $u = f(x_1, \dots, x_n)$ and let each $x_j = x_j(t_1, \dots, t_m)$. Then

$$\frac{\partial u}{\partial t_j} = \sum_{k=1}^n \frac{\partial u}{\partial x_k} \cdot \frac{\partial x_k}{\partial t_j}.$$

Under the Einstein convention, it can be written as

$$u_{t_j} = u_{x_k}(x_k)_{t_j} = u_k(x_k)_j.$$

The product rule on vector-valued functions.

1.

$$\frac{\partial(f \cdot g)}{\partial t} = \frac{\partial f}{\partial t} \cdot g + f \cdot \frac{\partial g}{\partial t}$$

2.

$$\frac{\partial(f \times g)}{\partial t} = \frac{\partial f}{\partial t} \times g + f \times \frac{\partial g}{\partial t}$$

Examples:

1. Prove that if $\|\alpha(t)\|^2 = 2$, then $\langle \alpha'(t), \alpha(t) \rangle = 0$.

2. Prove that

$$\frac{d(e^{At})}{dt} = Ae^{At} = e^{At}A.$$